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Gravito-inertial fields and relativity

H. G. L. COSTER and J. R. SHEPANSKI
School of Physics, The University of New South Wales, Sydney, Australia
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Abstract. Gravitational and inertial field equations of a Maxwellian form are established which lead to a consistent theory of gravitation. The new approach to dynamics arising from the field theory is in agreement with the theory of special relativity, while producing a fresh insight into some of the relativistic phenomena.

1. Introduction

On Einstein's theory of general relativity the gravitational field appears to occupy a very special position: unlike the very much stronger electromagnetic field, it is fully incorporated into the geometrical structure of space–time. In contrast with the special theory of relativity, the general theory, although widely accepted, has not borne many immediately applicable results.

Recently, Scott (1967) has shown that it is feasible to set up gravitational field equations based on a flat (Galilean) space–time by analogy with the electrodynamics of moving media. The present communication is concerned with another approach in which field equations are established in a Maxwellian form.

The new approach is capable of correctly predicting the precession of the perihelion of Mercury while the gravitational red shift and the bending of a light ray in a gravitational field can be obtained in the usual way. It is, therefore, consistent in these respects, as is Scott's, with the findings of the general theory.

It can be shown (Møller 1952) that the Maxwellian-type gravitational equations may be obtained, as an approximation, from the general theory of relativity on applying a linearization procedure. Basically, however, the new theory avoids the complications of the general theory and introduces instead, new fields to which we shall refer here as the *gravito-inertial* fields, in analogy with the electromagnetic fields.

In the present paper we shall demonstrate the consistency of our approach with the special theory and attempt to shed some new light on the interpretation of the variation of mass with velocity. Also, we shall verify, on the model theory, the applicability of the principle of equivalence and establish the possible existence of new gravitational inertial effects not normally accounted for by the theory of relativity.

2. The gravito-inertial field equations

The Maxwellian-type field equations, referred to in the introduction, are constructed here by postulating analogous roles for the field strengths $\mathscr{G}$ and $\mathscr{I}$, of the gravitational and inertial fields (see below), to the respective field strengths $\boldsymbol{E}$ and $\boldsymbol{H}$, of the electric and magnetic fields. We shall confine our considerations to the free space only.

The vectors $\mathscr{G}$ and $\mathscr{I}$ are, therefore, taken to satisfy the following field equations:

$$\begin{aligned} &\text{(i)}\ \nabla\times\mathscr{G} = \delta_0\frac{\partial\mathscr{I}}{\partial t} &&\text{(iii)}\ \nabla\cdot\mathscr{G} = -\frac{1}{\alpha_0}\rho_{\mathrm{g}} \\ &\text{(ii)}\ \nabla\times\mathscr{I} = \boldsymbol{j}_{\mathrm{g}}-\alpha_0\frac{\partial\mathscr{G}}{\partial t} &&\text{(iv)}\ \nabla\cdot\mathscr{I} = 0. \end{aligned} \tag{1}$$

This is in complete analogy with the Maxwellian equations for the electromagnetic field.

If we assume that the analogue of the electric charge here is the gravitational mass of a particle, whether moving or not, relative to the observer, then we may at once identify $\mathscr{G}$ as the gravitational attractive force per unit mass. This means that the gravitational force on a test particle of mass m', due to a gravitational field strength $\mathscr{G}$, is given by

$$\boldsymbol{F} = m'\,\mathscr{G}. \tag{2}$$

The quantity α_0, the gravitic permittivity of free space, is then given, in terms of the universal constant of gravitation γ, as

$$\alpha_0 = \frac{1}{4\pi\gamma} \sim 1{\cdot}19\times 10^9 \text{ m.k.s.} \tag{3}$$

The quantity $\boldsymbol{G}$

$$\boldsymbol{G} \equiv \alpha_0\,\mathscr{G} \tag{4}$$

may then be called the gravitic displacement or gravitic induction.

The quantity ρ_g is simply the local gravitational mass density and the negative sign in (1(iii)) is self-evident from the invariantly attractive nature of $\mathscr{G}$.

The inertial field strength $\mathscr{I}$ is associated with the particle momentum density $\boldsymbol{\pi}$ through the identification

$$\boldsymbol{\pi} \equiv \nabla\times\mathscr{I} + \alpha_0\frac{\partial\,\mathscr{G}}{\partial t} = \boldsymbol{j}_g. \tag{5}$$

$\boldsymbol{\pi}$ has therefore a component due to the analogue of the magnetomotive force and a component due to the gravitic displacement current.

The force on a particle of mass m', moving with a velocity $\boldsymbol{v}$ in a system in which there exists a gravito-inertial field, is then given by

$$\boldsymbol{F} = m'(\mathscr{G} + \boldsymbol{v}\times\boldsymbol{I}) \tag{6}$$

where

$$\boldsymbol{I} \equiv \delta_0\mathscr{I}. \tag{7}$$

$\boldsymbol{I}$ may be called the inertial induction, while the quantity δ_0 is the inertial permeability of free space.

The choice of signs in equations (1(i)) and (1(ii)) is determined from the following:

(i) The divergence of (1(ii)) gives

$$\frac{\partial\rho_g}{\partial t} + \nabla\cdot\boldsymbol{j}_g = 0 \tag{8}$$

a continuity-type equation identifying $\boldsymbol{j}_g$ with the gravitational mass current density.

(ii) Equations (1(ii)) also yield

$$\frac{1}{c'^2}\frac{\partial^2\,\mathscr{G}}{\partial t^2} - \nabla^2\,\mathscr{G} = \boldsymbol{S}_g \tag{9}$$

and

$$\frac{1}{c'^2}\frac{\partial^2\mathscr{I}}{\partial t^2} - \nabla^2\mathscr{I} = \boldsymbol{S}_i$$

where

$$c' \equiv (\delta_0\alpha_0)^{-1/2}. \tag{10}$$

These equations represent wave propagation of the $\mathscr{G}$ and $\mathscr{I}$ fields with the speed c'. $\boldsymbol{S}_g$ and $\boldsymbol{S}_i$ are source functions for these inhomogeneous wave equations, and are given by

$$\boldsymbol{S}_g = \frac{1}{\alpha_0}\left(\nabla\rho_g + \frac{1}{c'^2}\frac{\partial\boldsymbol{j}_g}{\partial t}\right) \tag{11}$$

and

$$\boldsymbol{S}_i = \nabla\times\boldsymbol{j}_g.$$

The equations (9) and (10) are in accord with the current ideas on gravitational propagation. It will be shown in the next section that c' also represents the limiting value of the speed of a particle, and hence $c' = c$.

3. Mass and gravito-inertial interactions

Let us consider a material particle moving with a velocity $\boldsymbol{u}$ relative to an observer O (cf. figure 1). The present location of the particle from O is given by the position vector $\boldsymbol{r}_0$, while $\boldsymbol{r}$ is the corresponding retarded position vector.

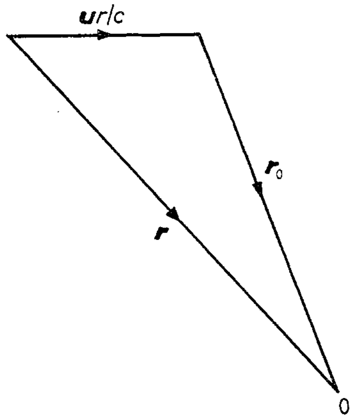

Figure 1.

Let the gravitational mass of the particle, as measured in its own rest frame, be m_0.

Taking due account of the finite speed c' of propagation for the gravito-inertial field, we can at once write down the *retarded* scalar and vector potentials at 0 due to the particle (cf. the electromagnetic theory):

$$\psi_g \equiv \frac{1}{4\pi\alpha_0} m_0 \frac{1}{r - \boldsymbol{r} \,.\, \boldsymbol{u}/c'} \tag{12}$$

$$\boldsymbol{\Psi}_i \equiv +\frac{\delta_0}{4\pi} m_0 \frac{\boldsymbol{u}}{r - \boldsymbol{r} \,.\, \boldsymbol{u}/c'}. \tag{13}$$

The corresponding gravitic field strength $\mathscr{G}$ and inertial induction $\boldsymbol{I}$ are then given by

$$\mathscr{G} = -\nabla\psi_g + \frac{\partial}{\partial t}\boldsymbol{\Psi}_i \tag{14a}$$

and

$$\boldsymbol{I} = \nabla \times \boldsymbol{\Psi}_i. \tag{14b}$$

Equations (14) with (12) and (13) yield

$$\mathscr{G} = -\frac{1}{4\pi\alpha_0} m_0 \frac{\boldsymbol{r}_0}{r_0{}^3} \left\{1 - \frac{(\boldsymbol{u} \times \hat{\boldsymbol{r}}_0)^2}{c'^2}\right\}^{-3/2} \left(1 - \frac{u^2}{c'^2}\right) \tag{15}$$

and

$$\boldsymbol{I} = +\frac{\delta_0}{4\pi} \frac{m_0}{r_0{}^3} (\boldsymbol{u} \times \boldsymbol{r}_0) \left\{1 - \frac{(\boldsymbol{u} \times \hat{\boldsymbol{r}}_0)^2}{c'^2}\right\}^{-3/2} \left(1 - \frac{u^2}{c'^2}\right). \tag{16}$$

(Note again the analogy with the electromagnetic treatment.)

The gravitational field, as determined by the observer at O, does, therefore, correspond to a locally evaluated effective gravitational mass:

$$m = m_0 \left\{1 - \frac{(\boldsymbol{u} \times \hat{\boldsymbol{r}}_0)^2}{c'^2}\right\}^{-3/2} \left(1 - \frac{u^2}{c'^2}\right). \tag{17}$$

Interpretation of equation (17) will be postponed until the value of c' has been determined.

In view of the relationship (5), it is natural to define the *effective* inertial mass density ρ_i by writing

$$\boldsymbol{\pi} = \nabla \times \mathscr{I} + \alpha_0 \frac{\partial \mathscr{G}}{\partial t} \equiv \rho_i \boldsymbol{u} \tag{18}$$

if the mass distribution contributing to ρ_i is moving locally with the velocity $\boldsymbol{u}$ relative to the observer.

From (16) we then have

$$\mathscr{I} = \frac{1}{\delta_0} \boldsymbol{I} = -\alpha_0(\boldsymbol{u} \times \mathscr{G}) \tag{19}$$

so that, when the velocity $\boldsymbol{u}$ is constant for the whole mass distribution,

$$\boldsymbol{\pi} = -\alpha_0 \nabla \times (\boldsymbol{u} \times \mathscr{G}) + \alpha_0 \frac{\partial \mathscr{G}}{\partial t} = -\alpha_0 (\nabla \cdot \mathscr{G}) \boldsymbol{u} = \rho_g \boldsymbol{u}. \tag{20}$$

Hence, on comparing (20) with (18), we conclude that, at every point of the distribution,

$$\rho_g = \rho_i.$$

Thus the integral of ρ_i over the volume in question, yielding the *effective* (*inertial*) *mass* of the distribution, must, on this theory, be identical with the corresponding effective (gravitational) mass, in accordance with the principle of equivalence.

An insight into the very concept of the inertia of a particle can be obtained by observing that when a particle is accelerated an induced gravitic field arises which is given by

$$\mathscr{G} = -\frac{\partial \boldsymbol{\Psi}_i}{\partial t}.$$

This induced field produces a reaction on the particle, so that an external force must be present in order to maintain the accelerated motion.

Substitution of $\boldsymbol{\Psi}_i$ from equation (13) shows that this reaction force for a slowly moving particle is proportional to the acceleration.

4. Directional aspects of the gravito-inertial field and limiting particle speed

It can be readily seen from (17) that the apparent mass deduced locally from the observed gravitic field is not only a function of velocity, but also depends on the direction of observation as long as $\boldsymbol{u} \neq 0$. There are two cases of particular interest:

(i) When

$$\boldsymbol{r}_0 \perp \boldsymbol{u}$$

$$m = m_0 \left(1 - \frac{u^2}{c'^2}\right)^{-1/2} \tag{21a}$$

and

(ii) when

$$\boldsymbol{r}_0 \parallel \boldsymbol{u}$$

$$m = m_0 \left(1 - \frac{u^2}{c'^2}\right). \tag{21b}$$

It is thus seen from (21*a*) that the apparent gravitational (and inertial) mass, as viewed in a direction normal to its velocity, increases with $\boldsymbol{u}$ in a manner entirely analogous to that predicted by the special theory of relativity. From (21*b*) it follows that, in the longitudinal direction, the gravitational and inertial interactions *decrease* as $u \to c'$. This is a very important result since it shows that any such interaction, which may be responsible for increasing the velocity of the particle in this direction, does, in fact, approach zero as $u \to c'$. This automatically imposes an upper limit to the particle speed.

The same upper limit c' on the value of $\boldsymbol{u}$ is also imposed by the catastrophic increase in the particle mass m, viewed transversely (cf. equation (21)).

The special theory of relativity gives an upper limit to the particle speed, equal to c, the invariant propagation speed of all electromagnetic signals *in vacuo*.

If the dynamics of the particle is preserved in all inertial frames, then the field equations (1) must retain their form in all such systems. The field propagation speed c' then becomes the *invariant* upper limit to the particle speed.

Since there can exist only one such *invariant* limiting speed, we conclude that

$$c = c' = (\alpha_0 \delta_0)^{-1/2}. \tag{22}$$

The result (22) has two important consequences:

(*a*) The relationship (21*a*) becomes identical with that predicted by the special theory. This suggests a new interpretation for the relativistic mass increase. This is discussed in detail below.

(*b*) Since c is now the speed of propagation of the gravito-inertial signals in free space, an agreement with the current ideas on the subject of propagation of gravitational fields is assured, while a more detailed description of such propagation becomes feasible on our model.

We must conclude from (15) and (16) that as $u \to c$, the gravito-inertial field tends to be confined to the transverse plane passing through the particle. Since the electromagnetic field of a charged particle behaves in a similar fashion, it is clear that there is no way in which one could directly interact longitudinally with a particle whose speed nears c.

5. Dynamical mass, momentum and energy relations

In a previous section it was shown that the local evaluation of a particle mass is dependent on the direction of observation (equation (17)).

The integral form of equation (1(iii)) gives the following expression for the Gauss law of the gravitic field:

$$m = \int \rho_g \, dV = -\alpha_0 \int \nabla \cdot \mathscr{G} \, dV = -\alpha_0 \int \mathscr{G} \cdot d\boldsymbol{S}. \tag{23}$$

The total dynamical mass of the field distribution can then be obtained by performing the integration in (23) over a surface surrounding the particle in its *retarded* position. This requires a knowledge of $\mathscr{G}$ in terms of this retarded position. For a given retarded position the value of $\boldsymbol{r}_0$, which gives the present position of the particle from the point of observation at any instant, is dependent on the direction of observation. The functional relationship between the present position and the direction of observation, for a given retarded position, can be calculated using the invariance of the propagation of the gravito-inertial field. The results also correspond to a Lorentz length contraction in the direction of motion and could be obtained in a manner entirely analogous to the corresponding problem for electromagnetic fields.

Thus, if R_0 is the value of the radial distance r_0 of the particle when in the transverse direction, it is easy to show that

$$R_0 = r_0 \left\{ \left(1 - \frac{(\boldsymbol{u} \times \hat{\boldsymbol{r}}_0)^2}{c^2}\right) \Big/ \left(1 - \frac{u^2}{c^2}\right) \right\}^{1/2} \tag{24}$$

where r_0 is the radial distance (of the present position) measured in any particular direction.

With the transformation (24) and using (15), the integral in (23) yields

$$m = m_0 \left(1 - \frac{u^2}{c^2}\right)^{-1/2} \tag{25}$$

It should be pointed out that, if the expression for $\mathscr{G}$ in terms of the present position from any one given observation point (i.e. equation (15)) is substituted directly into (23), the integration yields just m_0. This is not surprising since in doing so it is intrinsically assumed that all observers on a sphere of radius r_0 see the particle in the same present position for a given retarded position. This is clearly only possible when $\boldsymbol{u} = 0$.

In the derivation of equation (17), from (14*a*), giving the mass variation with velocity, it is possible to identify two distinct contributions. These arise, respectively, from the negative gradient of the scalar potential ψ_g and the time variation of the vector potential $\boldsymbol{\Psi}_i$.

Thus the integral in equation (23) consists of two terms. The total dynamical mass m given in equation (25) may therefore be written as

$$m = m_\psi + m_\Psi \tag{26}$$

where

$$m_\psi = m_0\left(1-\frac{u^2}{c^2}\right)^{-3/2}$$

and

$$m_\Psi = \frac{-m_0u^2}{c^2}\left(1-\frac{u^2}{c^2}\right)^{-3/2}. \tag{27}$$

Hence the effect of the time variation of the vector potential $\boldsymbol{\Psi}_i$ is to reduce the rate of mass increase with speed.

It is interesting to note that m_ψ is identical with what was sometimes referred to in the past as the longitudinal mass. Its origin lies in the propagation of the gravitic field.

The total momentum can be obtained in a similar fashion from equations (20) and (23) and using the transformation (24). Thus

$$\begin{aligned} \boldsymbol{P} = \int \boldsymbol{\pi}\,\mathrm{d}V &= \int \rho_g \boldsymbol{u}\,\mathrm{d}V \\ &= -\alpha_0 \int (\mathscr{G}\,.\,\mathrm{d}\boldsymbol{S})\boldsymbol{u} \\ &= m_0\left(1-\frac{u^2}{c^2}\right)^{-1/2}\boldsymbol{u} = m\boldsymbol{u} \end{aligned} \tag{28}$$

which is the normal relativistic expression for the particle momentum.

Equation (28), of course, yields directly the total energy E

$$E = mc^2. \tag{29}$$

The results predicted by the theory, which were derived without recourse to the results of the special theory of relativity, are in full agreement with that theory. The new approach, however, gives a completely new interpretation of relativistic phenomena.

References

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